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Overlapping layers in a gas lubrication problem

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Abstract

This investigation considers the operation of a one-dimensional squeeze slider bearing, operating under so-called slip flow conditions. At large values of the bearing and squeeze numbers, slide and squeeze effects in the pressure field in the bearing's interior are confined to narrow layer regions at the leading and trailing edges of the bearing, with squeeze layers at both edges, while a single slide layer is located at the trailing edge, coincident with the squeeze layer there. The structure of these layers is analyzed for appropriately large values of these parameters, and the method of matched expansions is applied to construct a leading-order expression for the pressure field throughout the bearing gap. It is found that squeeze effects at the trailing edge are transmitted to the bearing's (layer-free) interior via the (thicker) slide layer.

Keywords: Squeeze slider bearing; Gas lubrication; Slip flow; Singular perturbations

1. Introduction

This paper analyzes the operation of a gas-lubricated bearing in which the bearing gap width profile displays an oscillatory time variation, as well as spatial variation, occurring as a result of the upper surface of the bearing (see Fig. 1) being held stationary, while the lower surface, displaying periodic corrugations, is moved unidirectionally parallel to itself with constant speed. Such a situation arises in the modelling of the action of a computer disc memory, in which the upper surface represents the flying reader head, held stationary, while the lower surface represents the rotating disc itself, which is striated, to help eliminate electrostatic sticking. In such cases, the structure of the pressure field in the bearing gap displays two features (a) slide, characterized by the bearing number, Λ, and arising from the translational motion and (b) squeeze, characterized by the squeeze number, σ, and arising from the oscillatory component. This modelling has been considered using numerical techniques in

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a number of articles, and for a range of these parameters e.g., [2,3]. When the bearing is infinitely wide, so that it may be regarded as one dimensional, and side leakage effects are ignored, such analysis reveals that, at large values of the bearing and squeeze numbers, slide and squeeze effects in the pressure field in the bearing's interior are confined to narrow layer regions at the leading and trailing edges, with squeeze layers at both edges, and a single slide layer located at the trailing edge, coincident with the squeeze layer there.

For appropriate ratios of Λ and σ, the slide layer at the trailing edge contains the squeeze layer there, so that access to the 'outer' (layer-free) region in the bearing gap from the squeeze region can only be obtained via the wider slide layer; i.e., the slide layer defines a transition region, where slide effects dominate. This seriously affects the matching process when matching techniques are applied to construct an overall approximating expression for the pressure throughout the whole bearing gap.

2. Governing equations

The geometry for the general squeeze slider bearing is as shown in Fig. 1. Gaseous lubricant flows in the *bearing gap* contained between upper and lower surfaces; and bounded laterally by the planes $X = 0$, L and $Z = \pm\frac{1}{2}B$. Thus, the bearing gap is of length (or depth) L and breadth B. Typically, a varying lubricant gap is generated by uniform translation of one of the surfaces parallel to the positive X-axis; with the other surface remaining stationary. Thus, the bearing gap width at points (X, Z) and times T is characterized by a positive function $H(X, Z, T)$. If the lubricating gas is rarefied, slip flow conditions prevail; and the modified Reynolds equation governing the pressure

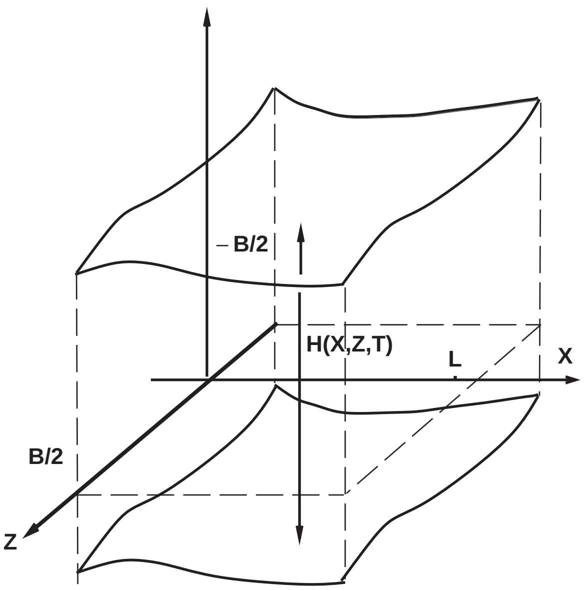

Fig. 1. Geometry for the general squeeze slider bearing.

$P(X,Z,T)$ in the bearing gap becomes

$$\frac{\partial}{\partial X}\left[H^3P\left(1+\frac{6\lambda_a}{PH}\right)\frac{\partial P}{\partial X}\right]+\frac{\partial}{\partial Z}\left[H^3P\left(1+\frac{6\lambda_a}{PH}\right)\frac{\partial P}{\partial Z}\right]$$
$$=6\mu U\frac{\partial}{\partial X}(PH)+12\mu\frac{\partial}{\partial T}(PH), \tag{1}$$

where $U>0$ is the speed of the uniform translation referred to above, μ is the (constant) gas viscosity; and λ_a is a measure of the gas molecular mean free path in the gap.

It is also assumed that the pressure reaches the (constant) ambient value P_a at the lateral bearing boundaries, so that boundary conditions relevant to (1) above are

$$P(0,Z,T)=P(L,Z,T)=P(X,\pm\tfrac{1}{2}B,T)=P_a. \tag{2}$$

The boundary value problem (1), (2) above for the pressure P may be rendered dimensionless, by choosing dimensionless quantities x,z,h, p and t, defined by

$$\left.\begin{aligned}&X=Lx,\qquad Z=Bz,\\&H(X,Z,T)=H_0h(x,z,t),\\&P(X,Z,T)=P_a\,p(x,z,t),\\&T=\omega^{-1}t,\end{aligned}\right\}, \tag{3}$$

where L, B and P_a are as above, while H_0 is typical (constant) value of H on $0\leqslant X\leqslant L$, $-\frac{1}{2}B\leqslant Z\leqslant\frac{1}{2}B$. Characteristically, since H depends on time T as well, H_0 might be some form of time-average of H for some prescribed (X,Z) value—e.g., $(0,0)$. Further, since the temporal variation of H is usually oscillatory, ω may be chosen as a characteristic frequency associated with that variation.

With choices (3), problem (1),(2) converts to the dimensionless form

$$\varepsilon^2\frac{\partial}{\partial x}\left[h^3p\left(1+\frac{6k}{hp}\right)\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[h^3p\left(1+\frac{6k}{hp}\right)\frac{\partial p}{\partial z}\right]$$
$$=\varepsilon^2\Lambda\frac{\partial}{\partial x}(hp)+\varepsilon^2\sigma\frac{\partial}{\partial t}(hp), \tag{4}$$

$$\begin{aligned}&p(x,\pm\tfrac{1}{2},t)=1,\quad 0\leqslant x\leqslant 1,\\&p(0,z,t)=p(1,z,t)=1,\quad -\tfrac{1}{2}\leqslant z\leqslant\tfrac{1}{2},\end{aligned} \tag{5}$$

where the dimensionless *breadth parameter*, ε, is defined by

$$\varepsilon=\frac{B}{L}, \tag{6}$$

the *bearing number*, Λ, is defined by

$$\Lambda = \frac{6\mu UL}{P_a H_0^2}, \tag{7}$$

the *squeeze number*, σ, is defined by

$$\sigma = \frac{12\mu\omega B^2}{P_a H_0^2}, \tag{8}$$

and the *Knudsen number*, k, is defined by

$$k = \frac{\lambda_a}{H_0}. \tag{9}$$

The non negative dimensionless parameters Λ, σ and k describe the bearing operation. Λ measures the relative longitudinal speed of the bearing surfaces, with high speed operation corresponding to $\Lambda \to \infty$. σ is a measure of the frequency of transverse variations in the profile $h(x,z,t)$; so, rapid transverse variation corresponds to $\sigma \to \infty$. k measures the degree of rarefaction of the gas lubricant and characterizes the degree of gas slip at the bearing walls. Typically, k may be regarded as an order one quantity, with $k \approx 0$–2. Thus, $k > 0$ denotes so-called *slip flow*, with $k = 0$ corresponding to *nonslip* flow, while higher k-values indicate successively higher degrees of slip.

For a large range of bearing geometries, h does not depend on z, i.e., $h = h(x,t)$. For such wedge bearings, with the assumption of negligible side leakage of lubricant, the limit $\varepsilon \to \infty$ converts problem (4), (5) to that of obtaining the solutions $p(x,t,\Lambda,\sigma,k)$ of the nonlinear partial differential equation

$$\frac{\partial}{\partial x}\left[h^3 p\left(1 + \frac{6k}{hp}\right)\frac{\partial p}{\partial x}\right] = \Lambda\frac{\partial}{\partial x}(hp) + \sigma\frac{\partial}{\partial t}(hp), \tag{10}$$

on $0 \leqslant x \leqslant 1, t \geqslant 0$, that meet the two-point boundary conditions

$$p(0,t,\Lambda,\sigma,k) = p(1,t,\Lambda,\sigma,k) = 1. \tag{11}$$

For its complete solution, the nonlinear boundary value problem (10), (11) also requires an initial condition, i.e., p needs to be prescribed at $t = 0$. However, the present investigation is concerned with solutions of (10), (11) as a result of the ongoing (typically oscillatory) time variation of h, i.e., after initial transitions generated by initial p values have died away. This is usually found to occur rapidly in applications [1].

DiPrima [1] applied perturbation techniques based on $\Lambda \to \infty$, $\sigma \to \infty$ to a suitably linearized form of the problem defined above in the nonslip ($k = 0$) case, to obtain approximate expressions for the pressure field, assuming appropriate gap profiles $h(x,t)$. He also considered the nonlinear problem, but resorted to numerical methods for parts of analysis. He found that, the pressure displayed layers of thickness $O(\sigma^{-1/2})$ ("squeeze layers") at the ends $x = 0, 1$, and one $O(\Lambda^{-1})$ layer (a "slide layer") at the trailing edge $x = 1$. The influence of these layers throughout the rest of the bearing then depended on the relative orders of magnitude of large Λ and σ. To date, this analysis has not been extended to the fully nonlinear slip flow case. Interest in the problem lapsed, but has been revived much more recently by White [2,3], who has applied analytical and numerical methods to an analysis of the operation of a computer disc memory, where one surface (the lower) is striated, to eliminate electrostatic sticking. While his analysis does address slip flow, it lacks the completeness of [1].

This paper considers a case of some interest, noted in [1], namely that for which Λ and σ are both large, but for which

$$\frac{\Lambda^2}{\sigma} \to 0 \quad \text{as} \quad \Lambda \text{ and } \quad \sigma \to \infty. \tag{12}$$

In this case, squeeze effects dominate those of slide, and there are two coincident layers at the trailing edge $x = 1$, with (12) ensuring that the slide layer is thicker than the squeeze one.

Leading order matching will be used to construct an approximation for the pressure field valid uniformly on $0 \leqslant x \leqslant 1$, and all times t.

3. Solution away from layers—the outer solution

In the region away from any layers, Eq. (10) is expected to determine the pressure. The rearrangement

$$\left(\frac{1}{\Lambda}\right)^2 \left(\frac{\Lambda^2}{\sigma}\right) \frac{\partial}{\partial x}\left[h^3 p \left(1 + \frac{6k}{hp}\right)\frac{\partial p}{\partial x}\right] = \left(\frac{1}{\Lambda}\right)\left(\frac{\Lambda^2}{\sigma}\right)\frac{\partial}{\partial x}(hp) + \frac{\partial}{\partial t}(hp), \tag{13}$$

displaying combinations of the gauge functions $(1/\Lambda)$ and (Λ^2/σ), implies that in such a region, the pressure p may be expanded as

$$p(x,t,\Lambda,\sigma,k) = p_0(x,t,k) + \left(\frac{1}{\Lambda}\right) p_1(x,t,k) + \left(\frac{\Lambda^2}{\sigma}\right) p_2(x,t,k) + \dots \ . \tag{14}$$

Note that (14) implies that $(1/\Lambda) \gg (\Lambda^2/\sigma)$. This will be assumed for now, with other possibilities to be mentioned below.

Substituting (14) into (13) and equating terms of like orders in small quantities yields

$$\frac{\partial}{\partial t}(hp_0) = \frac{\partial}{\partial x}(hp_0) = 0, \tag{15}$$

with corresponding equations for higher order coefficients in (14). Conditions (**??**) then imply that

$$p_0(x,t,k) = \frac{A}{h(x,t)}, \tag{16}$$

where A is a *constant*; giving the leading-order term of expansion (14).

When, in fact, $(1/\Lambda) \ll (\Lambda^2/\sigma)$, the opposite of that assumed above, the roles of p_1 and p_2 are interchanged; while outcome (16) for p_0 is unchanged.

Note that since

$$p_0(0,t,k) = \frac{A}{h(0,t)}; \qquad p_0(1,t,k) = \frac{A}{h(1,t)},$$

no choice of the *constant* A can make p_0 satisfy either one of the boundary conditions (11). Thus, as expected, layers occur adjacent to both end-points $x = 0$ and 1.

4. The squeeze layers

As observed above, layers adjacent to $x=0$ and 1 of thickness $O(\sigma^{-1/2})$ arise from the squeezing component of the motion. These can be analyzed using a matched expansions approach. Thus, in the layer at $x=0$, the local variable

$$\xi = x\sqrt{\sigma} \tag{17}$$

may be introduced. In terms of the variables ξ and t, the differential equation (10) becomes

$$\frac{\partial}{\partial \xi}\left[h^2(hP+6k)\frac{\partial P}{\partial \xi}\right] = \frac{\Lambda}{\sqrt{\sigma}}\frac{\partial}{\partial \xi}(hP) + \frac{\partial}{\partial t}(hP), \tag{18}$$

where $P(\xi, t, \Lambda, \sigma, k)$ is the transformed form of p; and h is evaluated at $(\xi/\sqrt{\sigma}, t)$.

Eq. (18) is assumed valid in the squeeze layer region adjacent to $x=0$, where $\xi = O(1)$. In this region, a layer expansion

$$P = P_0(\xi,t,k) + \frac{\Lambda}{\sqrt{\sigma}}P_1(\xi,t,k) + \frac{1}{\sqrt{\sigma}}P_2(\xi,t,k) + \cdots \tag{19}$$

is assumed to represent the pressure P, and substituting into (18) gives the differential equation

$$h(0,t)^2\frac{\partial}{\partial \xi}\left[(h(0,t)P_0+6k)\frac{\partial P_0}{\partial \xi}\right] = \frac{\partial}{\partial t}(h(0,t)P_0) \tag{20}$$

for the leading term $P_0(\xi,t,k)$.

Since the expansion (19) is assumed valid up to $x=\xi=0$, P_0 must satisfy

$$P_0(0,t,k) = 1, \tag{21}$$

while leading-order matching with the 'outer' solution (16) implies the condition

$$P_0(\infty,t,k) = p_0(0,t,k) = \frac{A}{h(0,t)}. \tag{22}$$

For given A, the function $P_0(\xi,t,k)$ may now be found by finding the transient free solution of (20), subject to the two boundary conditions (21), (22). As noted by DiPrima [1], this calculation must be carried out using an appropriate numerical technique.

The problem now becomes one of determining the value of the constant A. This may be achieved by an elementary extension of the argument presented in [1]. If it is noted that $h(x,t)$ is periodic, of period T (usually 2π); then both of p_0 and P_0, should inherit this periodicity. In the light of this, consider the differential equation (20). This may be rearranged as

$$\frac{1}{2}\frac{\partial^2}{\partial \xi^2}[h(0,t)(h(0,t)P_0+6k)^2] = \frac{\partial}{\partial t}(h(0,t)P_0),$$

and integration of this across one cycle gives, with the periodicity of the right side,

$$\int_0^T h(0,t)[h(0,t)P_0(\xi,t,k)+6k]^2\,dt = \alpha\xi + \beta, \tag{23}$$

where T is the period of h, and α, β are constants.

For matching to be achievable, the right-hand side of (23) must remain finite as $\xi \to \infty$. This implies $\alpha = 0$, and then (23) implies that

$$\int_0^T h(0,t)[h(0,t)P_0(\xi,t,k) + 6k]^2\,\mathrm{d}t,$$

is constant, for all $0 \leqslant \xi < \infty$.

This yields

$$\int_0^T h(0,t)\left[h(0,t)\lim_{\xi\to\infty} P_0(\xi,t,k) + 6k\right]^2 \mathrm{d}t = \int_0^T h(0,t)[h(0,t) + 6k]^2\,\mathrm{d}t,$$

and (22) gives

$$(A+6k)^2 \int_0^T h(0,t)\,\mathrm{d}t = \int_0^T h(0,t)[h(0,t) + 6k]^2\,\mathrm{d}t, \tag{24}$$

which determines A.

Note that since the pressure is positive, the positive root of (24) is taken.

If the standard notation

$$\langle f \rangle = \frac{1}{T}\int_0^T f(t)\,\mathrm{d}t,$$

is adopted, (24) may be rewritten

$$(A+6k)^2 = \frac{\langle h(0,t)[h(0,t)+6k]^2\rangle}{\langle h(0,t)\rangle}. \tag{25}$$

Analogous arguments for the squeeze layer at $x = 1$ in terms of the layer variable

$$\zeta = (1-x)\sqrt{\sigma} \tag{26}$$

lead to the leading term $\tilde{P}_0(\zeta,t,k)$ in the squeeze layer at $x = 1$ being given as the transient free solution of the equation

$$h(1,t)^2\frac{\partial}{\partial\zeta}\left[(h(1,t)\tilde{P}_0 + 6k)\frac{\partial\tilde{P}_0}{\partial\zeta}\right] = \frac{\partial}{\partial t}(h(1,t)\tilde{P}_0), \tag{27}$$

satisfying the condition

$$\tilde{P}_0(0,t,k) = 1. \tag{28}$$

If it is assumed that $\tilde{P}_0$ is to match directly to the 'outer' solution p_0, the condition

$$\tilde{P}_0(\infty,t,k) = \frac{A}{h(1,t)}, \tag{29}$$

must apply. However, if the arguments applied above to Eq. (20) are applied to (27) (with the assumption of periodicity of $\tilde{P}_0$), the condition

$$(A+6k)^2 = \frac{\langle h(1,t)[h(1,t)+6k]^2\rangle}{\langle h(1,t)\rangle}, \tag{30}$$

analogous to (25), is arrived at.

It is clear that, for arbitrary $h(x,t)$, the values of A determined by (25) and (30) are incompatible. Thus, direct matching between the squeeze layers and the outer expansion is *not* achievable. There must be an intermediate region (layer) in which slide effects, governed by $\Lambda \to \infty$, become important. Since, for a purely sliding bearing, the layer is located at $x = 1$, it might be expected that, in the present case, this layer is located there as well; and the matching from squeeze layer at $x = 1$ to the outer region must be accomplished via this slide layer region.

5. The slide layer

The previous section established the presence of a layer adjacent to $x = 1$, of thickness $O(\Lambda^{-1})$, that arises from the sliding motion of the bearing. This layer coincides with the squeeze layer of that section, there, and condition (12) ensures that this slide layer is thicker than the squeeze layer.

To analyze this layer, the local variable

$$\eta = (1 - x)\Lambda, \tag{31}$$

is introduced. In terms of this variable, Eq. (10) converts to

$$\left(\frac{\Lambda^2}{\sigma}\right)\frac{\partial}{\partial \eta}\left(\tilde{h}^2(\tilde{h}\tilde{p} + 6k)\frac{\partial \tilde{p}}{\partial \eta}\right) = -\left(\frac{\Lambda^2}{\sigma}\right)\frac{\partial}{\partial \eta}(\tilde{h}\tilde{p}) + \frac{\partial}{\partial t}(\tilde{h}\tilde{p}), \tag{32}$$

where $\tilde{p}(\eta,t,\Lambda,\sigma,k) \equiv p(1 - \Lambda^{-1}\eta,t,\Lambda,\sigma,k)$, $\tilde{h} \equiv h(1 - \Lambda^{-1}\eta,t)$.

If a local layer expansion

$$\tilde{p}(\eta,t,\Lambda,\sigma,k) = \tilde{p}_0(\eta,t,k) + +\frac{\Lambda^2}{\sigma}\tilde{p}_1(\eta,t,k) + \cdots, \tag{33}$$

is substituted into (32) and the limit $\Lambda^{-1} \to 0$ invoked, the coefficients $\tilde{p}_0, \tilde{p}_1$ may be shown to satisfy the equations

$$\frac{\partial}{\partial t}(h(1,t)\tilde{p}_0) = 0, \tag{34}$$

and

$$\frac{\partial}{\partial \eta}\left[h(1,t)^2(h(1,t)\tilde{p}_0 + 6k)\frac{\partial \tilde{p}_0}{\partial \eta}\right] + \frac{\partial}{\partial \eta}(h(1,t)\tilde{p}_0) = \frac{\partial}{\partial t}(h(1,t)\tilde{p}_1). \tag{35}$$

Eq. (34) implies that the quantity $h(1,t)\tilde{p}_0$ is independent of time, i.e., depends only on η.

If Eq. (35) is averaged over one time period, and the periodicity of h and $\tilde{p}_0$ (and hence $\tilde{p}_0, \tilde{p}_1$) noted, there results

$$\langle h(1,t)\rangle\frac{\partial}{\partial \eta}\left[(h(1,t)\tilde{p}_0 + 6k)\frac{\partial}{\partial \eta}(h(1,t)\tilde{p}_0)\right] + \frac{\partial}{\partial \eta}(h(1,t)\tilde{p}_0) = 0, \tag{36}$$

an ordinary differential equation for $h(1,t)\tilde{p}_0$, which has the general solution

$$h(1,t)\tilde{p}_0 + (B + 6k)\ln|h(1,t)\tilde{p}_0 - B| = -\frac{\eta}{\langle h(1,t)\rangle} + C, \tag{37}$$

where B and C are (as yet unknown) constants.

This function $\tilde{p}_0(\eta,t)$ is assumed to represent, to leading order, the pressure p between the squeeze layer at $x = 1$ and the outer region to the bearing.

6. Matching and the composite expansion

With the value of the constant A given by (25), the leading order term P_0 in the squeeze layer at $x=0$ matches with that of the 'outer' expansion, given by (16).

Now, leading order matching between the slide layer at $x=1$ and the outer expansion gives, from (37) and (16),

$$B = A. \tag{38}$$

Putting $\eta = 0$ in (37) gives, with (38),

$$h(1,t)\tilde{p}_0(0,t,k) + (A+6k)\ln|h(1,t)\tilde{p}_0(0,t,k) - A| = C \tag{39}$$

which implies, that, since A and C are constants,

$$h(1,t)\tilde{p}_0(0,t,k) = \text{constant} = D, \quad \text{say}. \tag{40}$$

Now reconsider the calculation leading to (30), at $\eta = 0$ $(x=1)$. This gives,

$$\int_0^T h(1,t)\left[h(1,t)\lim_{\zeta\to\infty}\tilde{P}_0(\zeta,t,k) + 6k\right]^2 \mathrm{d}t = \int_0^T h(1,t)[h(1,t)+6k]^2\,\mathrm{d}t,$$

and if the squeeze layer expansion is to match, to leading order, to the slide layer expansion, so that

$$\lim_{\zeta\to\infty}\tilde{P}_0(\zeta,t,k) = \tilde{p}_0(0,t,k),$$

this becomes, with (40)

$$\int_0^T h(1,t)[D+6k]^2\,\mathrm{d}t = \int_0^T h(1,t)[h(1,t)+6k]^2\,\mathrm{d}t,$$

or

$$(D+6k)^2 = \frac{\langle h(1,t)[h(1,t)+6k]^2\rangle}{\langle h(1,t)\rangle}, \tag{41}$$

defining the value of D.

With D given by (41), C is given, from (39), by

$$C = D + (A+6k)\ln|D-A| \tag{42}$$

in terms of A and D.

Thus, the leading term of the squeeze layer expansion at $x=1$ is given as the transient free solution of (27) that satisfies (28) and the condition

$$\tilde{P}_0(\infty,t,k) = \frac{D}{h(1,t)}, \tag{43}$$

which is distinct from (29), since $D \neq A$.

With A and D determined as above, a leading-order expression $p_c(x,t,\Lambda,\sigma,k)$ (the "composite expansion") for the pressure throughout the whole of the bearing at all times may be obtained from the rule

$$p_{\mathrm{c}} = P_0(x\sqrt{\sigma},t,k) + p_0(x,t,k) + \tilde{p}_0((1-x)\Lambda,t,k)$$
$$+ \tilde{P}_0((1-x)\sqrt{\sigma},t,k) - \frac{A}{h(0,t)} - \frac{A}{h(1,t)} - \frac{D}{h(1,t)} + \cdots, \tag{44}$$

i.e., the sum of the leading order expansions minus their "common parts", where P_0, p_0, $\tilde{p}_0$ and $\tilde{P}_0$ are as defined earlier.

7. Discussion

The analysis of the above section has identified subregions within the lubricating film ($0 \leqslant x \leqslant 1$) in which particular effects dominate throughout the operation of the bearing. Thus, squeeze effects dominate in thin layer regions at the leading ($x = 0$) and trailing ($x = 1$) edges; while slide effects dominate in a thicker layer region adjacent to the trailing edge. In the bearing's interior ($0 < x < 1$), away from layer regions, neither squeeze nor slide has any leading-order effect.

Within these layer regions, the dominant terms of the governing differential equation (10) may be identified; and application of appropriate perturbation methods has yielded the leading-order expansion (44), which is a plausible approximation to the pressure field in the bearing gap profile over all of $0 \leqslant x \leqslant 1$ and for ongoing time. For a given profile function $h(x,t)$, the 'outer' solution, p_0, given by (16) is known explicitly; and although the slide layer term $\tilde{p}_0$ of (44) is given in implicit form (Eq. (37)), it is readily evaluated using such standard software as Maple or Mathematica. Only the squeeze layer contributions, P_0 and $\tilde{P}_0$ must be calculated numerically by solving a nonlinear diffusion equation. However, such numerical solvers as PDEase make this calculation fairly routine.

It is of some value to consider the structure of the pressure field as represented by (44) for a particular bearing profile function $h(x,t)$. Thus, the profile

$$h(x,t) = 1 - 0.5x + 0.2\sin t, \tag{45}$$

will be used. This is of the general form of such profiles as defined in [1]. Since the squeeze layers must be constructed numerically, their structure will only be sketched here.

Away from the squeeze layers, the pressure may be assumed to be represented by

$$p_0(x,t,k) + \tilde{p}_0((1-x)\Lambda, t, k) - \frac{A}{h(1,t)}, \tag{46}$$

a partial composite expansion incorporating slide effects.

Fig. 2 shows the variation of this along the bearing, together with its evolution in time. The x-variation clearly shows the slide effects characteristic of converging steady state profiles, namely an increase in pressure towards the trailing edge. The oscillatory development of this profile in time is quite clear, arising from the periodicity of the profile. Of course, the boundary conditions at $x=0,1$ are not met (though nearly so at $x = 0$) at all times; and the disrepancy is to be compensated for by the squeeze layer at the leading and trailing edge.

As noted above, the leading terms $P_0, \tilde{P}_0$ in the squeeze layer expansions must be constructed numerically. Fig. 3 displays one consequence of this. This shows the variation of the function $P_0(\xi,t)$ at $\xi = 1$ as time evolves, for the profile (45) as computed with PDEase, and compares it with its limiting value

$$\frac{A}{h(0,t)}$$

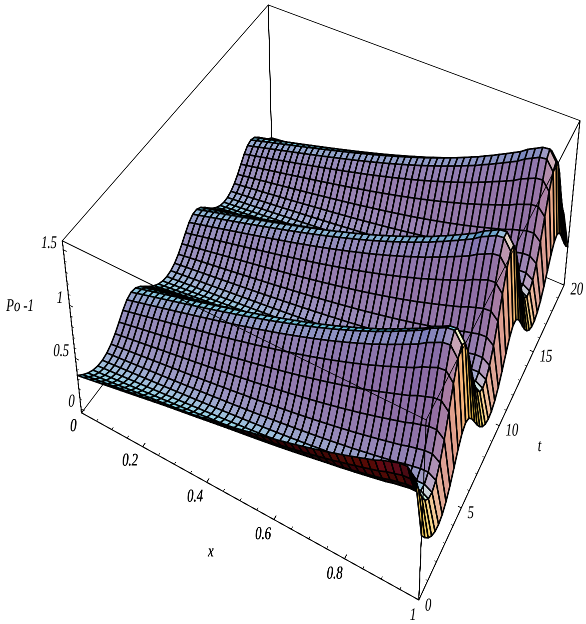

Fig. 2. Evolution of the pressure profile on $t \geqslant 0$, with squeeze layers deleted, as given by Eq. (46), for $\Lambda = 20, \sigma = 1600$, and $k = 0$.

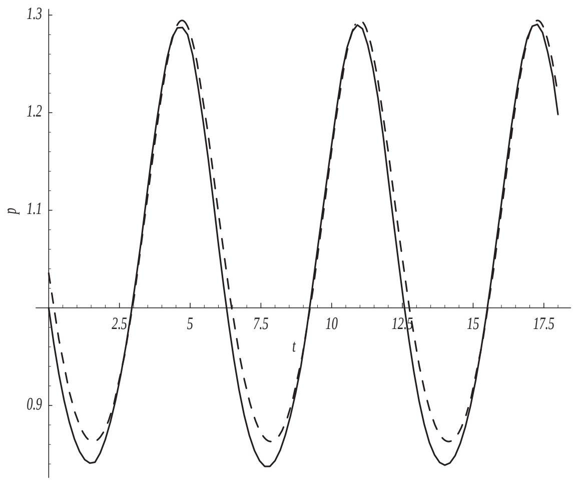

Fig. 3. Convergence of the numerically generated squeeze layer at leading edge, for the profile (45) at $\xi = 1$, for $\Lambda = 20, \sigma = 1600$, and $k = 0$.

as given in (22). The initial value $P_0(\xi, 0, k) = 1$ was chosen, with $k = 0$. While there is some discrepancy at $t = 0$, it is clear that very soon, transients have died out, and the function P_0 is relatively close to the limiting (time varying) value. This behavior is anticipated in the squeeze layer at $x = 1$, as well.

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